Name of college - S.S. college, J-Bad

PAGE NO: 1

DATE: 17-08-2020.

Deptt - Mathematics

Topic - The shortest distance between two lines, Intersection of three planes

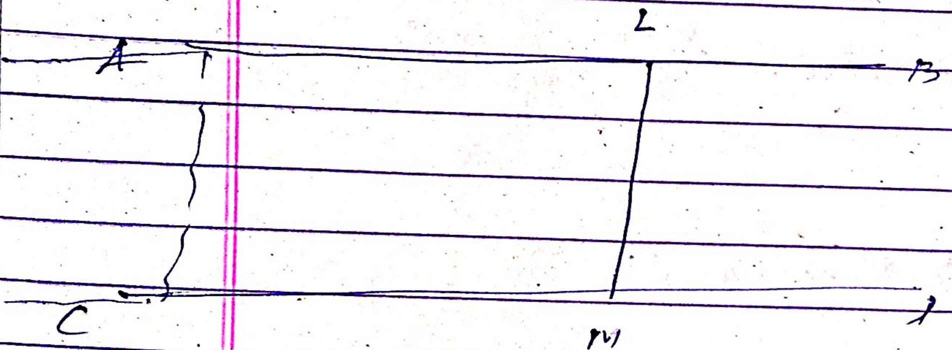
Time - 1:00 P.M to 1:45 PM

Date - 17-08-2020. Class - B.Sc I (PASS+HONS)

BY - Dr. Shree Nath Sharma

Study material :->

Shortest distance between two lines lies along the line meeting the two lines at right angles



Let AB and CD be the given two lines and LM is the line which intersect the two lines AB and CD at right angles. Thus LM is the shortest distance between AB and CD

If A be on AB and C be on CD

then LM is the projection of AC on LM

and so LM = AC cos θ

where θ = angle between LM and AC

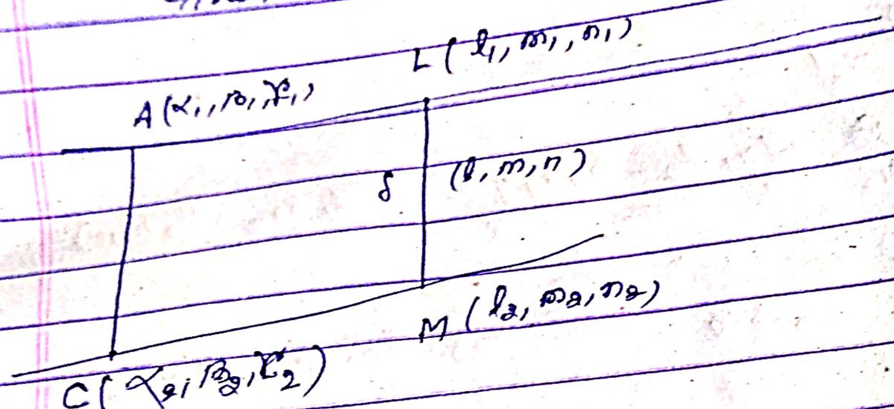
thus LM < AC

and LM is the shortest distance

DATE 17.08.2021

Between two lines AB and CD

Magnitude and the equation of the shortest distance between two given st. lines



Let AB and CD be two st. lines

Where equation of AB is

$$\frac{x-\alpha_1}{l_1} = \frac{y-\beta_1}{m_1} = \frac{z-\gamma_1}{n_1}$$

and equation of CD is

$$\frac{x-\alpha_2}{l_2} = \frac{y-\beta_2}{m_2} = \frac{z-\gamma_2}{n_2}$$

Where $(\alpha_1, \beta_1, \gamma_1)$ is co-ordinate of A

$(\alpha_2, \beta_2, \gamma_2)$ is co-ordinate of B

Let $LM = \delta$ be the shortest distance between AB and CD

Since LM is perpendicular to AB and CD both. Let (l, m, n) be the direction cosine of LM

$$\Rightarrow \quad l l_1 + m m_1 + n n_1 = 0$$

$$l l_2 + m m_2 + n n_2 = 0$$

On solving

$$\frac{l}{m_1 n_2 - m_2 n_1} = \frac{m}{n_1 l_2 - n_2 l_1} = \frac{n}{l_1 m_2 - l_2 m_1} = K$$

$\Rightarrow l = K(m_1 n_2 - m_2 n_1)$

$m = K(n_1 l_2 - n_2 l_1)$

$n = K(l_1 m_2 - l_2 m_1)$

Def. $l^2 + m^2 + n^2 = 1$

$\Rightarrow K^2 \left[(m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - n_2 l_1)^2 + (l_1 m_2 - l_2 m_1)^2 \right] = 1$

$\Rightarrow K^2 \sin^2\theta = 1$

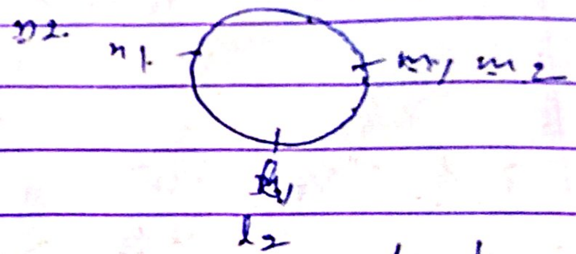
$K^2 = \frac{1}{\sin^2\theta} = \csc^2\theta$ as $\sin\theta = \Sigma(m_1 n_2 - m_2 n_1)^2$

$\Rightarrow K = \csc\theta$

$\therefore l = (m_1 n_2 - m_2 n_1)\csc\theta$

$m = (n_1 l_2 - n_2 l_1)\csc\theta$

$n = (l_1 m_2 - l_2 m_1)\csc\theta$



Since shortest distance is perpend
to both lines. AC is projection o
LM

Let projection of AC on LM whose
direction cosines are (l, m, n)

$$f = \frac{(\alpha_2 - \alpha_1)l + (\beta_2 - \beta_1)m + (\gamma_2 - \gamma_1)n}{\sin\theta}$$

$$= \frac{(\alpha_2 - \alpha_1)(m_1 n_2 - m_2 n_1) + (\beta_2 - \beta_1)(n_1 l_2 - \ldots + (\gamma_2 - \gamma_1)(l_1 m_2 - l_2 m_1)}{\sin\theta}$$

DATE 17-08-20

$$\delta = \begin{vmatrix} \alpha_2-\alpha_1 & \beta_2-\beta_1 & \gamma_2-\gamma_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}$$

$\sin\theta$

Now it Remains to Find the Equation of LM

, Since LM is the intersection of planes ALM and CLM

∴ the Equation of plane ALM

$$\begin{vmatrix} x-\alpha_1 & y-\beta_1 & z-\gamma_1 \\ l_1 & m_1 & n_1 \\ l & m & n \end{vmatrix} = 0$$

and CLM is

$$\begin{vmatrix} x-\alpha_2 & y-\beta_2 & z-\gamma_2 \\ l_2 & m_2 & n_2 \\ l & m & n \end{vmatrix} = 0$$

And these represent the line LM.

Remarks → Shortest distance LM = 0

⟹ the lines are coplanar

i.e two st lines are coplanar, the Distance between them = 0.

PAGE NO: 5
DATE: 17-08-202

Problem 1. Find the shortest distance between the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ and obtain its equation.

Solution :→ Let the shortest distance meet the lines in P and Q respectively

Here equation is 1st line is

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = r$$

∴ Co-ordinate of any point P on it is

$(2r+1, 3r+2, 4r+3)$

and ~~Co-ordi~~ equation of 2nd line is

$$\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5} = r_1$$

then Co-ordinate of any point Q on it is

$(3r_1+2, 4r_1+4, 5r_1+5)$

∴ Direction ratio of PQ are

$3r_1+2-2r-1, \quad 4r_1+4-3r-2, \quad 5r_1+5-4r-3$

i.e. $3r_1-2r+1 \quad 4r_1-3r+2 \quad 5r_1-4r+2$ — (I)

Since PQ is perpendicular to 1st line

$\Rightarrow 2(3r_1-2r+1) + 3(4r_1-3r+2) + 4(5r_1-4r+2) = 0$ — (II)

also PQ is perpendicular to 2nd line

∴ $3(3r_1-2r+1) + 4(4r_1-3r+2) + 5(5r_1-4r+2) = 0$ — (III)

∴

PAGE NO 06
DATE 17-08-2020

From (11) $29r - 88r_1 - 16 = 0$

$38r - 50r_1 - 21 = 0$

using cross multiplication

$$\frac{r}{38\times21 - 16\times50} = \frac{r_1}{-16\times38 + 21\times29} = \frac{1}{-29\times50 + 38\times88}$$

$$\frac{r}{798 - 800} = \frac{r_1}{-608 + 609} = \frac{1}{-1450 + 1444}$$

$$\Rightarrow \frac{r}{-2} = \frac{r_1}{1} = \frac{1}{-6}$$

$\therefore r = 1/3 \quad r_1 = -1/6$

Co-ordinates of P are $x_1 = 1 + 2r = 1 + 2/3 = 5/3$

$y_1 = 2 + 3r = 2 + 3\times\frac{1}{3} = 3$

$z_1 = 3 + 4r = 3 + 4\times\frac{1}{3} = \frac{13}{3}$

Co-ordinates of Q are $x_2 = 2 + 3r_1 = 2 + 3(-1/6) = 3/2$

$y_2 = 4 + 4r_1 = 4 + 4(-1/6) = \frac{10}{3}$

$z_2 = 5 + 5r_1 = 5 + 5(-1/6) = \frac{25}{6}$

$\therefore PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$

$= \left(\frac{3}{2} - 5/3\right)^2 + \left(\frac{10}{3} - 3\right)^2 + \left(\frac{25}{6} - \frac{13}{3}\right)^2$

$= \left(-\frac{1}{6}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{6}\right)^2$

$= \frac{1}{36} + \frac{1}{9} + \frac{1}{36}$

$= \frac{1+4+1}{36} = \frac{1}{6}$

$PQ = \frac{1}{\sqrt{6}}$

Equation of PQ

$$\frac{x - 5/3}{-1} = \frac{y - 3}{2} = \frac{z - \frac{13}{3}}{-1}$$